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Control with Random Access Wireless Sensors

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***Abstract*— We consider multiple sensors randomly accessing a shared wireless medium to transmit measurements of their respective plants to a controller. To mitigate the packet collisions arising from simultaneously transmitting sensors, we appropriately design the sensor access rates. This is posed as an optimization problem, where the total transmit power of the sensors is minimized, and control performance for all control loops needs to be guaranteed. Control performance of each loop is abstracted as a desired expected decrease rate of a given Lyapunov function. By establishing an equivalent convex optimization problem, the optimal access rates are shown to decouple among sensors. Moreover, based on the Lagrange dual problem, we develop an easily implementable distributed procedure to find the optimal sensor access rates.**

I. Introduction

The abundance of wireless sensing devices in modern control environments, e.g., smart buildings and urban infrastructure, creates a need for sharing the available wireless medium in an easily implementable manner that also provides control performance guarantees. The prevalent approach for sharing a communication medium in networked control systems has been centralized scheduling. Scheduling may be static, specifying that sensors transmit in some periodic sequence predesigned to meet control objectives [1]–[3]. Deriving optimal scheduling sequences is recognized as a hard combinatorial problem [4]. Scheduling may also be dynamic, where a central authority decides which device accesses the medium at each time step based on, e.g., plant state information [5], [6] or wireless channel conditions [7].

In contrast to centralized approaches, we are interested in a decentralized mechanism (random access) for sensors to share the wireless medium. Each sensor independently and randomly decides whether to transmit plant state measurements over the channel to a controller. This mechanism is easy to implement as it does not require predesigned sequences of how sensors access the medium or a central authority to take scheduling decisions. However packet collisions can occur from simultaneously transmitting sensors, resulting in control performance degradation. Sensor access rates need to be designed to mitigate these effects.

Control under random access communication mechanisms has drawn limited attention, to the best of our knowledge. Comparisons between different medium access mechanisms for networked control systems and the impact of packet collisions have been considered in [8]–[11], including random access and related Aloha-like schemes (where after a packet collision the involved sensors wait for a random time interval and retransmit). Stability conditions under packet collisions were examined in [12]. In contrast to these works, our goal is to design the medium access mechanism so that desired control performance is guaranteed. Related work appears in [13], which instead considers Aloha-like retransmission policies that lead to stability. Besides closed loop control, sensor transmission over collision channels for optimal remote estimation is considered recently in [14].

We consider multiple control loops over a shared wireless channel (Section II). We design the rate at which the sensor of each loop should access the channel to transmit to a common access point/controller, in order to ensure control performance for all loops. We employ a Lyapunov-like control performance requirement, motivated by our work on centralized scheduling [7]. Each control system is abstracted via a given Lyapunov function which is desired to decrease at predefined rates in expectation, due to the random packet losses and collisions on the shared medium. These control requirements are shown to be equivalent to a minimum packet success rate on each link.

We examine the design of sensor access rates that satisfy the Lyapunov control performance requirements and minimize the average transmit power of the sensors. After reformulating this as a convex optimization problem, a characterization of the optimal sensor access rates is established based on Lagrange duality (Section III). This characterization reveals an intuitive decoupled form; each sensor should access the channel at a rate proportional to the desired control performance of its corresponding control loop, and inverse proportional to its transmit power and the aggregate collision effect it causes on all other control loops. Similar decoupled structures are known in the context of random access wireless networks [15]–[18], where the relevant quantities of interest are data rates on links or general utility objectives. In our case in contrast we focus on packet success rates for desired control system performance.

In Section IV we derive a decentralized procedure converging to the optimal access rates, which has an interpretation of optimizing the dual problem. The procedure is easy to implement as it does not require the sensors to coordinate among themselves, or to know what other sensors try to achieve. The procedure just relies on the common access point to provide dual variables to the sensors via the reverse channel. We conclude with a numerical example and some remarks (Sections V, VI).

II. Problem Formulation

We consider a wireless control architecture where m independent plants are controlled over a shared wireless medium. Each sensor i ($i = 1, 2, ..., m$) measures and transmits the output of plant i to an access point responsible for computing

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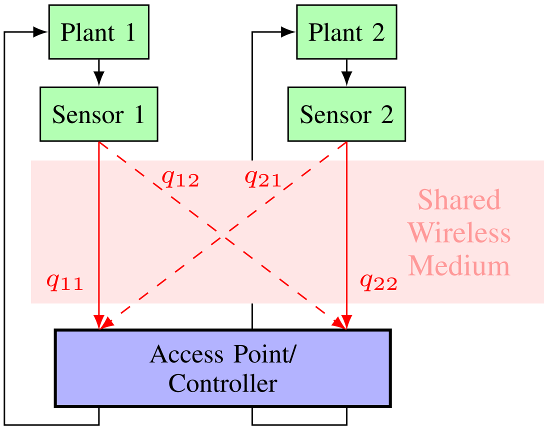

Fig. 1. Random access architecture over a shared wireless medium for $m = 2$ control loops. Each sensor randomly decides whether to transmit to a common access point computing the plant control inputs. When only sensor i is transmitting on the shared medium, the message is successfully decoded with probability q_{ii}. When another sensor j transmits at the same slot, there is a probability q_{ji} of causing a collision at sensor i's transmission, rendering i's packet lost. The goal is to design the rate at which every sensor accesses the medium in order to guarantee control performance for all control loops.

the plant control inputs (Fig. 1). We assume the dynamics for all m control systems are fixed and controllers have been pre-designed. Our goal is to design the wireless communication aspects of the problem, in particular a random access mechanism for sensors to transmit over the shared medium.

The evolution of each control system i depends on whether a transmission occurs on respective link i at time k or not, denoted by a random indicator variable $\gamma_{i,k} \in \{0, 1\}$. We suppose each system evolution is described by a switched linear time invariant model of the form

$$x_{i,k+1} = \begin{cases} A_{c,i}\, x_{i,k} + w_{i,k}, & \text{if } \gamma_{i,k} = 1 \\ A_{o,i}\, x_{i,k} + w_{i,k}, & \text{if } \gamma_{i,k} = 0 \end{cases}. \qquad (1)$$

Here $x_{i,k} \in \mathbb{R}^{n_i}$ denotes the state of control system i at each time k, which may in general include both plant and controller states [19]. At a successful transmission the system dynamics are described by the matrix $A_{c,i} \in \mathbb{R}^{n_i \times n_i}$, where 'c' stands for closed-loop, and otherwise by $A_{o,i} \in \mathbb{R}^{n_i \times n_i}$, where 'o' stands for open-loop. We assume that $A_{c,i}$ is asymptotically stable, implying that if system i successfully transmits at each slot the state evolution of $x_{i,k}$ is stable. The open loop matrix $A_{o,i}$ may be unstable. The additive terms $w_{i,k}$ model an independent (both across time k and across systems i) identically distributed (i.i.d.) noise process with mean zero and covariance $W_i \succeq 0$.

Communication in our random access framework takes place in time slots. At every time k each sensor i randomly and independently decides to access the channel with some constant probability $\alpha_i \in [0, 1]$. The vector of sensor access rates $\alpha \in [0, 1]^m$ is the design variable in our system. The sensor measurements transmitted in the shared wireless medium may be dropped due to two reasons. First, if only some sensor i transmits at a time slot, decoding at the access point/controller may fail due to noise added to the transmitted signal or fading effects of the wireless channel [20]. We assume that successful decoding occurs with some known constant positive probability $q_{ii} \in (0, 1]$.

The second reason for packet drops is interference due to other sensors transmitting at the same time slot as sensor i does. In particular if another sensor j transmits, a collision occurs on sensor i's link with some probability q_{ji}, leading to packet i being lost. The values q_{ji} are assumed to be known constants, for simplicity positive $q_{ji} \in (0, 1]$. The case where some sensors do not interfere with each other is similarly handled – see Remark 3. This collision model has been recently considered in [18] and subsumes: i) the conservative case where simultaneous transmissions certainly lead to collisions ($q_{ji} = 1$) usually considered in control literature, e.g., [12], [13], ii) the case where simultaneously transmitted packets are not always lost ($q_{ji} < 1$), e.g., due to the capture phenomenon [21], and iii) the asymmetric case where different sensors j, ℓ interfere differently on link i, e.g., due to their spatial configuration.

Given that sensor j randomly decides to transmit with probability α_j, the probability that link i is affected by sensor j equals the product $\alpha_j\, q_{ji}$. To sum up, the combined effect from all sensors on packet success at link i yields

$$\mathbb{P}(\gamma_{i,k} = 1) = \alpha_i\, q_{ii} \prod_{j \neq i} \Big[1 - \alpha_j q_{ji} \Big]. \qquad (2)$$

This expression states that the probability of system i in (1) closing the loop at time k equals the probability that transmission i is successfully decoded at the receiver, multiplied by the probability that no other sensor $j \neq i$ is causing collisions on ith link. The product in this expression is a consequence of the fact that all sensors independently decide to access the channel . In (2) the channel parameters q_{ji}, $i, j \in \{1, \dots, m\}$ are given, and the variables to be designed are the sensor access rates α.

The random packet success on link i modeled by (2) causes each control system i in (1) to switch in a random fashion between the open and closed loop modes of operation. As a result the access rate vector α to be designed affects the performance of all control systems. The following result characterizes, via a Lyapunov-like abstraction, a connection between control performance and the packet success rate – see our previous work in [7] or [22] for a proof.

Theorem 1 (Control performance abstraction)**.** *Consider a switched linear system i described by (1) with $\gamma_{i,k}$ being an i.i.d. sequence of Bernoulli random variables, and a quadratic function $V_i(x_i) = x_i^T P_i x_i$, $x_i \in \mathbb{R}^{n_i}$ with a positive definite matrix $P_i \in S^{n_i}_{++}$. Then the function decreases with an expected rate $\rho_i < 1$ at each step, i.e., we have*

$$\mathbb{E}\left[V_i(x_{i,k+1}) \,\middle|\, x_{i,k} \right] \leq \rho_i\, V_i(x_{i,k}) + Tr(P_i W_i) \qquad (3)$$

for all $x_{i,k} \in \mathbb{R}^{n_i}$, if and only if

$$\mathbb{P}(\gamma_{i,k} = 1) \geq c_i, \qquad (4)$$

where $c_i \geq 0$ is computed by the semidefinite program

$$c_i = \min\{\theta \geq 0 : \; \theta A_{c,i}^T P_i A_{c,i} + (1 - \theta) A_{o,i}^T P_i A_{o,i} \preceq \rho_i P_i\} \qquad (5)$$

The interpretation of the quadratic function $V_i(x_i)$ is that it acts as a Lyapunov function for the control system, guaranteeing not only stability but also performance – see Remark 1. When the loop closes the Lyapunov function of the system state decreases, while in open loop it increases, and (3) describes an overall decrease in expectation over the packet success.

In this paper we assume that quadratic Lyapunov functions $V_i(x_i)$ and desired expected decrease rates ρ_i are given for each control system. As in our previous work on centralized scheduling [7], they present a control interface for communication design over a shared wireless medium. We aim to design the sensor access rates α so that the Lyapunov functions *for all systems* i decrease *in expectation* at the desired rates $\rho_i < 1$ at any time k. By the above theorem, these control performance requirements correspond to necessary and sufficient packet success rates for each link. Hence we need to ensure that (4) holds for all links i.

Besides control performance, it is desired that the channel access mechanism makes an efficient use of the sensors' power resources. We model each sensor i using constant power $p_i > 0$ when transmitting. We pose then the design of sensor access rates α that minimize the total expected power expenditure $\sum_{i=1}^m \alpha_i\, p_i$ subject to the desired control performance (cf. (2), (3), (4)) for all plants as

$$\underset{\alpha \in \mathcal{A}}{\text{minimize}} \quad \sum_{i=1}^{m} \alpha_i p_i \tag{6}$$

$$\text{subject to} \quad c_i \leq \alpha_i\, q_{ii} \prod_{j \neq i} \Big[1 - \alpha_j q_{ji}\Big],\ i \in \{1, \dots, m\}. \tag{7}$$

For technical reasons we restrict attention to access rates α in a closed subset of the unit cube $[0, 1]^m$ of the form

$$\mathcal{A} = \prod_{i=1}^{m} \mathcal{A}_i, \quad \mathcal{A}_i = [\alpha_{i,\min}, \alpha_{i,\max}]. \tag{8}$$

This choice does not restrict the feasible set. Intuitively each sensor i can neither choose α_i too close to 0 otherwise it cannot meet its packet success requirement in (7), nor too close to 1 otherwise it causes significant packet collisions on other sensors. We assume that for all $\alpha \in \mathcal{A}$ the right hand side of the constraints (7) are strictly positive.

In the following section we proceed to characterize the optimal access rates α^*, by transforming the original non-convex problem (6)-(7) into an equivalent convex one. This way we reveal a simple decoupled structure, and exploit it later in Section IV to develop an easily implementable procedure to find these optimal access rates.

Remark 1. In this paper we consider communication design for control performance, in contrast to the more common problem of communication designs that guarantee stability, e.g. [1], [2], [13], [19]. The Lyapunov-like abstraction (3) provides a characterization of control performance, which also implies stability. If (3) holds at each time step k, the second moments of system states decay exponentially with rate $\rho_i < 1$ and remain bounded in the limit [7].

On the technical side, the Lyapunov performance approach specifies a convex region for the packet success rate in (4), which is easy to employ in our random access design in (7). On the contrary, a jump linear system of the form (1) is (mean square) stable if and only if the spectral radius of the non-symmetric matrix $\mathbb{P}(\gamma_i = 1) A_{c,i} \otimes A_{c,i} + \mathbb{P}(\gamma_i = 0) A_{o,i} \otimes A_{o,i}$ is less than unity [23]. This is in general a non-convex specification, hence it is unclear how to best examine stability in our random access framework. □

III. CONTROL-AWARE RANDOM ACCESS DESIGN

In this section we characterize the form of the optimal sensor access rates according to problem (6)-(7). This is a non-convex optimization problem because the functions appearing in the right hand side of the constraints (7) are not concave. However taking the logarithm at each side of (7) preserves the feasible set of variables by monotonicity. The logarithm of the product on the right hand side of (7) becomes a sum of logarithms, so that we can rewrite the optimal random access design problem equivalently as

$$\underset{\alpha \in \mathcal{A}}{\text{minimize}} \quad \sum_{i=1}^{m} \alpha_i p_i \tag{9}$$

$$\text{subject to} \quad \log(c_i) \leq \log(\alpha_i\, q_{ii}) \ + \sum_{j \neq i} \log\Big(1 - \alpha_j q_{ji}\Big),$$

$$\text{for all } i \in \{1, \dots, m\}. \tag{10}$$

This equivalent problem is convex in the access rate variables α, because the logarithm functions appearing in the constraints are concave. Hence the optimal rates α^* can be readily computed using standard convex optimization algorithms [24]. Apart from tractability, the convex reformulation permits the following theoretical characterization of the form of the optimal access rates.

Theorem 2 (Optimal control-aware random access)**.** *Consider the design of optimal control-aware sensor access rates in (6)-(7), and suppose that a strictly feasible solution exists. Then there exists a matrix of non-negative elements $\nu^* \in \mathbb{R}_+^{m \times m}$ such that the optimal sensor access rate α_i^* for each sensor $i \in \{1, \dots, m\}$ can be expressed as*

$$\alpha_i^* = \left[\frac{\nu_{ii}^*}{p_i + \sum_{j \neq i} \nu_{ji}^*}\right]_{\mathcal{A}_i}, \tag{11}$$

where $[.]_{\mathcal{A}_i}$ denotes the projection on the set $\mathcal{A}_i$ in (8).

This theorem surprisingly states that each sensor can select its access rate optimally in a simple decoupled way. That is because α_i^* in (11) only depends on parameters pertinent to system i, e.g., its transmit power p_i. In particular it is independent of what control performances other sensors are trying to achieve. All the information about the optimal rate in (11) is encoded in the matrix ν^*, which technically is the optimal Lagrange multiplier of an appropriately defined problem. Intuitively ν_{ii}^* can be thought as corresponding to the control performance requirement of closed loop i, and similarly ν_{ji}^* to the collision effect that sensor i has on another closed loop j. The optimal access rate for sensor i in (11) trades off the requirement on loop i and the collective negative effect ($\sum_{j\neq i} \nu_{ji}^*$) on all other control loops $j \neq i$. A high transmit power p_i also implies that sensor i should access the channel at a low rate α_i^* to limit expenditures.

Proof of Theorem 2. The original problem (6)-(7) is equivalent to the one in (9)-(10). For notational convenience we denote variables α_i by α_{ii} for $i = 1, \ldots, m$. We further introduce auxiliary variables α_{ji} in place of the variables α_j, $j \neq i$ at the right hand side of (10). To force the new variables α_{ji} to behave like α_j of the original problem (α_{jj} in the new notation) we introduce additional constraints $\alpha_{ji} \geq \alpha_{jj}$. Overall we formulate the auxiliary convex optimization problem

$$\underset{\alpha \in \mathcal{A}'}{\text{minimize}} \quad \sum_{i=1}^{m} \alpha_{ii} p_i \tag{12}$$

$$\text{subject to} \quad \log(c_i) \leq \log(\alpha_{ii}\, q_{ii}) + \sum_{j\neq i} \log\Big(1 - \alpha_{ji} q_{ji}\Big), \quad \text{for all } i \in \{1, \ldots, m\}, \tag{13}$$

$$\alpha_{jj} \leq \alpha_{ji}, \quad \text{for all } i \neq j \in \{1, \ldots, m\} \tag{14}$$

for an appropriate set $\mathcal{A}' \subset [0,1]^{m\times m}$ so that $\alpha_{ji} \in \mathcal{A}_j$ for all $i, j \in \{1, \ldots, m\}$. It is easy to argue that problem (12)-(14) is equivalent to (9)-(10), i.e., every feasible solution of one can be converted to a feasible solution of the other with equal objective.

We define the Lagrange dual problem of (12) by associating dual variables $\nu_{ii} \geq 0$ and $\nu_{ij} \geq 0$ with constraints (13), (14) respectively. The Lagrangian function is defined as

$$L(\alpha, \nu) = \sum_{i=1}^{m} \alpha_{ii} p_i + \sum_{i=1}^{m} \nu_{ii} \Big[\log(c_i) - \log(\alpha_{ii}\, q_{ii}) - \sum_{j\neq i} \log(1 - \alpha_{ji} q_{ji})\Big] + \sum_{i=1}^{m} \sum_{j\neq i} \nu_{ij} \Big[\alpha_{jj} - \alpha_{ji}\Big]. \tag{15}$$

By a rearrangement of the terms we get a form decoupled among primal variables,

$$L(\alpha, \nu) = \sum_{i=1}^{m} \Bigg\{ \Big[(p_i + \sum_{j\neq i} \nu_{ji})\, \alpha_{ii} - \nu_{ii} \log(\alpha_{ii}\, q_{ii})\Big] + \sum_{j\neq i} \Big[- \nu_{ii} \log(1 - \alpha_{ji} q_{ji}) - \nu_{ij} \alpha_{ji}\Big] + \nu_{ii} \log(c_i) \Bigg\}. \tag{16}$$

Strict feasibility of (6)-(7) implies strict feasibility for the convex problem (12)-(14), hence strong duality holds [24, Prop. 6.4.3]. The optimal primal α^* is a minimizer of the Lagrangian at the optimal dual point ν^*. Noting that the Lagrangian (16) is strictly convex in α, the minimizer α^* is unique and satisfies the first order condition $\frac{\partial L}{\partial \alpha}(\alpha, \nu^*) = 0$, subject to the box constraints $\alpha_{ji} \in \mathcal{A}_j$ for all $i, j \in \{1, \ldots, m\}$. By the decoupled Lagrangian in (16) we get

$$\frac{\partial L}{\partial \alpha_{ji}}(\alpha, \nu) = \begin{cases} (p_i + \sum_{j\neq i} \nu_{ji}) - \nu_{ii}/\alpha_i & \text{if } i = j \\ \nu_{ii} q_{ji}/(1 - \alpha_{ji} q_{ji}) - \nu_{ij} & \text{if } i \neq j. \end{cases} \tag{17}$$

This directly verifies the form of optimal α_i^* in (11). □

The decoupled structure of the optimal sensor access rates according to Theorem 2 relies on knowing the values ν^*. In the following section we develop a distributed iterative procedure, easily implementable in the architecture of Fig. 1, to obtain the desired ν^*.

Remark 2. The fact that the optimal sensor access rates can be decoupled among systems according to Theorem 2 is in accordance with known results for general random access communication networks [15]–[18]. The technical development in these works is similar to our optimization problem (6)-(7). The context differs however, since in general wireless networks the quantity of interest is the achieved throughput rates, fairness, or general utility functions, in contrast to the packet success rates used for closed loop control performance here. It is also worth noting that even though our collision model is also considered in [18], the exact decoupled form of Theorem 2 in (11) is novel. □

IV. Implementation of control-aware random access

We develop an iterative algorithm to determine the values ν^* which, according to Theorem 2, can be used by the sensors to select optimal channel access rates α^*. The algorithm is distributed and easily implementable in the sense that the common access point is responsible for maintaining tentative values ν which are sent to the sensors via the reverse channel in order for them to select appropriate access rates.

The steps of the iterative procedure are shown in Algorithm 1. At each iteration t, the access point/controller of Fig. 1 keeps a matrix of variables $\nu(t)$ and sends to each sensor i via the reverse channel the values $\nu_{ii}(t)$ and $\sum_{j\neq i} \nu_{ji}(t)$. The latter then selects its access rate $\alpha_i(t)$ in (18) as if the received values correspond to the optimal ones ν^* (cf. (11)). Then the access point updates the matrix to $\nu(t+1)$ to prepare for the next iteration.

Given the interpretation of $\nu(t)$ as Lagrange dual variables of the auxiliary problem (12)-(14), the update step in (22) is a step towards a subgradient direction $s(t) \in \mathbb{R}^{m\times m}$ of the dual function. This procedure converges to the optimal sensor access rates α^* as stated next.

Theorem 3 (Sensor access rates optimization)**.** *Consider the design of optimal control-aware sensor access rates in (6)-(7), and suppose that a strictly feasible solution exists. The iterations of Algorithm 1 with the stepsize in (22) satisfying $\sum_{t\geq 1} \varepsilon(t)^2 < \infty$, $\sum_{t\geq 1} \varepsilon(t) = \infty$, converge to the optimal access rates, i.e., $\alpha_i(t) \to \alpha_i^*$ for all $i = 1, \ldots, m$.*

Proof. The variables $\alpha_i(t), \alpha_{ji}(t)$ computed by Algorithm 1 in (18)-(19) are minimizers of the Lagrangian of the auxiliary problem (12)-(14) at the point $\mu(t)$ (cf.(17)). The matrix $s(t)$ computed by (20)-(21) corresponds to the slack of these minimizers in the auxiliary problem, hence is a subgradient of the dual function at $\mu(t)$ [24, Ch. 8.1]. Moreover the subgradient $s(t)$ is bounded due to the restriction $\alpha(t) \in \mathcal{A}'$ (cf.(8)), i.e., the logarithms in (20) are finite. Hence by subgradient optimization arguments [24, Prop. 8.2.6] for the selected stepsizes the dual variables $\nu(t)$ converge to the optimal ν^*. The variables $\alpha(t)$ also converge to the optimal sensor access rates α^* by continuity of (11). □

Algorithm 1 Distributed random access implementation

Initialize $\nu(0) \in \mathbb{R}_+^{m\times m}$ at the access point/controller, $t \leftarrow 0$

loop At period t

The access point/controller sends to each sensor i the values $\nu_{ii}(t)$, $\sum_{j\neq i}\nu_{ji}(t)$.

Each sensor i computes

$$\alpha_i(t) \leftarrow \left[\frac{\nu_{ii}(t)}{p_i + \sum_{j\neq i}\nu_{ji}(t)}\right]_{\mathcal{A}_i} \tag{18}$$

and for the rest of the period t it accesses the channel with rate $\alpha_i(t)$.

The access point/controller measures the access rates $\alpha_i(t)$ selected by all sensors $i = 1,\dots,m$ during the period and computes the auxiliary variables

$$\alpha_{ji}(t) \leftarrow \left[\frac{1}{q_{ji}} - \frac{\nu_{ii}(t)}{\nu_{ij}(t)}\right]_{\mathcal{A}_j} \quad \text{for all } j\neq i, \tag{19}$$

and the matrix $s(t) \in \mathbb{R}^{m\times m}$ with diagonal elements

$$s_{ii}(t) \leftarrow \log(c_i) - \log(\alpha_i(t)\, q_{ii}) - \sum_{j\neq i}\log\Big(1-\alpha_{ji}(t)q_{ji}\Big) \tag{20}$$

for all $i \in \{1,\dots,m\}$, and offdiagonal elements

$$s_{ij}(t) \leftarrow \alpha_j(t) - \alpha_{ji}(t) \tag{21}$$

for all $i,j \in \{1,\dots,m\}$, $i\neq j$.

The access point/controller computes the new matrix

$$\nu(t+1) \leftarrow \Big[\nu(t) + \varepsilon(t)\, s(t)\Big]_+ \tag{22}$$

where $[\ ]_+$ denotes the elementwise projection to the non-negatives $\mathbb{R}_+^{m\times m}$.

end loop

Apart from converging to the optimal operating point, Algorithm 1 is easily implementable in the wireless control architecture of Fig. 1. The sensors decide upon their access rates without coordination among themselves. Moreover they do not need to know global problem information, e.g., the specifications of the other coexisting control loops, or even how many other sensors are sharing the same wireless medium. Each sensor only needs to know the amount of collisions it causes on all other sensors collectively (captured by the value of the sum $\sum_{j\neq i}\nu_{ji}(t)$ in (18)).

The access point/controller on the other hand needs to know the packet success rates c_i required for control performance of each control loop i (cf. Theorem 1), as well as the channel collision pattern described by the values q_{ji}. At every iteration of the algorithm the access point additionally needs to know the sensor access rates $\alpha(t)$ selected by the sensors during this iteration, which can be either (i) computed since the access point knows all dual variables, or (ii) estimated using the empirical packet receptions, or (iii) sent by the sensors to the access point within the transmitted packets. Simulations of the algorithm are presented in the following section.

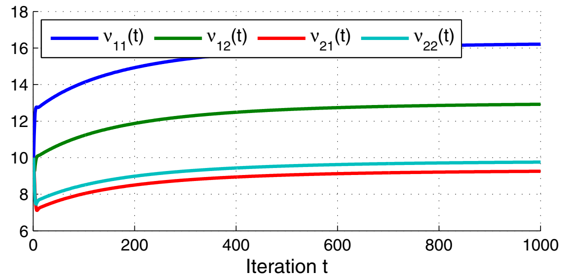

Fig. 2. Evolution of dual variables during the optimization algorithm. The elements of the matrix $\nu(t)$ converge to the optimal values ν^* required to obtain the optimal sensor access rates.

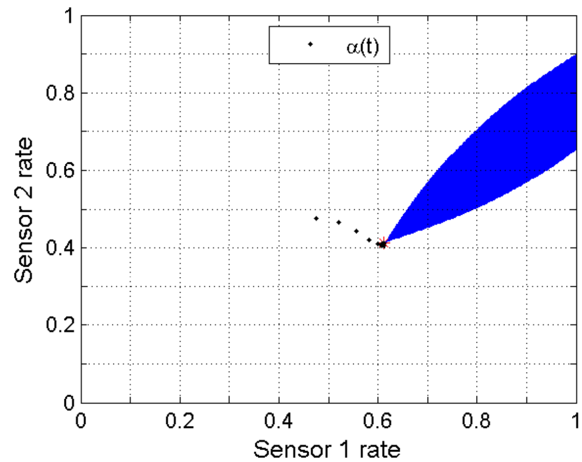

Fig. 3. Sensor access rates for the numerical example in Section V. The feasible set of sensor access rates that meet the control performance requirements of the two control systems is shown in shaded. After few iterations the access rates $\alpha(t)$ selected by the optimization algorithm converge close to the feasible point with the lowest utilization.

Remark 3. Our formulation can be modified for cases where some sensors do not interfere with others ($q_{ji} = 0$). Define the subset of sensors that cause collisions on link i as $I_i = \{j \neq i : q_{ji} > 0\}$, and conversely the subset of links that are affected by sensor i as $O_i = \{j\neq i : q_{ij} > 0\}$. The packet success probability in (2) is modified to include only interfering sensors $\prod_{j\in I_i}$. Similarly the optimal sensor access rates in (11) are modified to include the sum $\sum_{j\in O_i}\nu_{ji}^*$. Also in Algorithm 1 no 'coupling' variables α_{ji},ν_{ij} are needed when $j\notin I_i$. □

V. Numerical simulations

We present a numerical example of the random access design. As in Fig. 1 we consider $m = 2$ identical scalar control systems of the form (1), with open (unstable) and closed (stable) loop dynamics $A_{o,i} = 1.1$, $A_{c,i} = 0.4$ respectively. The two respective wireless sensors transmit to the access point/controller over a shared channel with success and collision parameters

$$\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22}\end{bmatrix} = \begin{bmatrix} 0.95 & 0.6 \\ 0.6 & 0.95\end{bmatrix} \tag{23}$$

i.e., in isolation 5% of the messages are dropped, and collisions happen with probability 60% in simultaneous transmissions. The transmit powers are taken equal $p_i = 1$. The systems and the channel are symmetric, but we model an asymmetric control performance requirement. A Lyapunov function $V_i(x) = x^2$ for each plant state is required to decrease with expected rates $\rho_1 = 0.75$ and $\rho_2 = 0.95$ respectively (cf. (3)). System 1 is more demanding, also shown by the required packet success rates $c_1 \approx 0.44$, $c_2 \approx 0.25$ of the two sensors, computed via (5).

We solve the random access design problem (6)-(7) by Algorithm 1, which as explained in Section IV solves the problem in the dual domain. The dual variables $\nu(t)$ of the algorithm converge as shown in Fig. 2. We also plot the evolution of the sensor access rates $\alpha(t)$ during the algorithm in Fig. 3, along with the set of all access rates that are feasible with respect to the control performance requirements (7). We observe that the sensor rate iterates $\alpha(t)$ start from infeasible values, and moves towards the extreme point of the feasible set with the lowest access rates, so that the power expenditure in (6) is minimized. In fact after only a few iterations of the algorithm the sensor access rates are very close to the optimal point, which is

$$\alpha_1^* \approx 0.61,\ \alpha_2^* \approx 0.41. \tag{24}$$

As expected, sensor 1 is accessing the shared channel at a higher rate than sensor 2 in order to achieve the more demanding control performance requirement of system 1. Moreover, both sensors access the channel at a rate higher than the necessary packet success rates, i.e., $\alpha_i^* > c_i$. This happens because the sensors need to counteract the effect of packet collisions, as well as packet drops due to decoding errors. In comparison to an ideal channel without collisions (but with packet drops) where each sensor would access the channel at rates c_i/q_{ii}, the increase in channel access is 47% for sensor 1, and 75% for sensor 2.

VI. Concluding remarks

We design a random access mechanism for sensors transmitting measurements of multiple plants over a shared wireless channel to a controller. The goal is to mitigate the effect of packet collisions from simultaneous transmissions and guarantee control performance for all control systems. Via a Lyapunov function abstraction, control performance is converted to required packet success rates of each closed loop. We show that the optimal rates at which sensors should transmit are decoupled among systems, and develop a distributed procedure to obtain them.

A caveat of the procedure is the required information exchange between sensors and the common access point/controller, introducing communication overhead. We have recently addressed this issue in [22] by a decentralized mechanism that does not require an access point. Future work includes the design of mechanisms able to adapt to, e.g. changes in the channel collision pattern and the control performance requirements, or to the admission of new control loops in the architecture. Future research also includes decentralized adaptation to plant states similar to, e.g., the scheduling in [5], the single link case in [20], or the remote estimation in [14].

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